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ANOMALOUS ROUGHENING IN A RANDOM SINE GORDON CHAIN

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Abstract

The appearance of solitons in a random sine-Gordon system is related to the roughening exponent, ζ, defined as the scaling exponent of the length of the ensemble average of the standard deviation of the height of the spatiotemporal profile. We show that the coherence of the ordered state appears to have three different regimes with well-defined crossover lengths. After the activation of solitons, there is a very interesting crossover from non-Kardar–Parisi–Zhang[6] behavior ($\zeta \sim 0.7$) to KPZ behavior ($\zeta \sim 0.5$); additionally, for sufficiently large scales, a crossover to a zero roughening exponent takes place. For the transient we calculate from flat initial conditions the common dynamic exponent ($\zeta/z \sim 0.9$) for all these regimes. This last result reveals that the surface grows faster than is predicted by the KPZ model. We point out the connection of our results to the Sneppen universality class[9] and discuss the crossover between the conditions of global dynamics and the conditions of local dynamics at different length scales in the stationary regime.

1. INTRODUCTION

The interplay between nonlinearity and noise results in fascinating physics in which, for example, growing interfaces develop fingers and overhangs.[1] The steady state of such interfaces remains self-affine and can be characterized by Hurst exponent.[2] Also, if the noise is strong enough, nonlinear solutions like solitons can be excited: statistical coherence appears as the random forcing is increased, and the system experiences a sharp crossover to an ordered regime characterized by a reduced number of degrees of freedom.[3] This process of noise-induced organization for the Sine-Gordon Chain has been characterized by a roughening exponent[4] as in dynamically driven interfaces in unstable and stable displacement in porous media.[5] The popular model of Kardar–Parisi–Zhang[6] has been very successful in explaining the roughening exponents measured in many surface growth models.[2] However, many experiments indicate the existence of at least two new universality classes.[7,8] In fact, in a recent model proposed by Sneppen,[9] the interface grows by a global search of optimal sites with quenched randomness, yielding long range correlated phenomena characterized by dynamic scaling exponents that differ from the KPZ universality class. Sneppen also pointed out the limitations of his model, arguing that global rules do not necessarily correspond to all regimes in experimental situations or models, and suggested a possible crossover from global rules below a certain scale to local rules for larger scales. This situation should be present in experiments on stable fluid displacement in porous media.[7]

2. THE RANDOM SINE GORDON CHAIN AND THE SOLITON GAS

In this paper we report numerical simulations using a straightforward discretization of the Sine-Gordon equation in a heat bath. The random and damped Sine-Gordon model is described by the following dimensionless equations:

$$\phi_{xx} - \phi_{tt} - \sin\phi = \alpha\phi_t - R(x, t)\,, \tag{1}$$

$$\langle R(x, t)\rangle = 0\,, \tag{2}$$

$$\langle R(x, t)R(x', t')\rangle = 2\alpha kT\delta(t - t')\delta(x - x')\,. \tag{3}$$

This model represents a chain of tightly coupled pendula interacting with a thermal reservoir. In our simulations the thermal noise term $R(x, t)$ is uncorrelated both in time and space. We use flat initial conditions ($\phi(x, 0) = \phi_t(x, 0) = 0$) and open boundary conditions ($\phi_x(0, t) = \phi_x(l, t) = 0$). In all the results to be presented in this article, $\Delta t = 0.035$ and $\Delta x = 0.039$.

The random and damped Sine-Gordon model has been previously considered by Büttiker and Landauer[10] in the overdamped limit and for a low level of noise. In this case the second derivative in time ϕ_{tt} is neglected, and the particles have negligible inertia in the presence of the friction α. Krug and Spohn[11] also included an additional term on the right side of Eq. (1) representing a uniform driving force F that assures the existence of the solitons.

In this paper we study the case when $F = 0$, but allow the intensity of the noise to be high. We would like to remark that in Ref. 11 a stochastic model of the kink gas (proposed in Ref. 10) was used to simulate the regime considered in this work. We also stress that our noise drive is "white" both in space and time and differs from the quenched type of noise used by Parisi[12] and by Sneppen.[9]

In our case, the particles have finite inertia. Therefore, spatiotemporal profiles above the roughening transition[3,4] are not just cases where $\phi(x, t)$ is such that the chain lies with segments in valleys of the potential $V = V_0(1 - \cos(\phi(x, t)))$ which are connected by *static* kinks or antikinks (overdamped limit). We have instead solitons that move over a finite length along the chain. This is because of the finite energy the solitons can acquire and dissipate. We use $V_0 = 1$ in our paper. This process of activation of solitons and their subsequent motion along the chain is not like Brownian motion, but maintains a spatiotemporal coherence, which gives rise to an anomalous roughening exponent ζ and an anomalous dynamic scaling β at small length scales. For sufficiently long scales however, we show below that the dynamics is like that of a kink gas. This occurs for decreasing length scales as the amplitude of the dissipation increases, i.e., α increases, approaching the overdamped regime. This picture is perfectly consistent with previous developments.[10,11]

3. ANOMALOUS ROUGHENING: TRANSIENT AND STATIONARY REGIMES

In Fig. 1 we present the standard deviation of the spatiotemporal profile,

$$\sigma(L,t) = \left\langle \left[\frac{1}{N_L} \sum_{i=1}^{N_L} (\phi_i - \bar{\phi})^2 \right]^{1/2} \right\rangle; \; \bar{\phi} = \frac{1}{N_L} \sum_{i=1}^{N_L} \phi_i \,, \tag{4}$$

as the system evolves from flat initial conditions ($\phi(x,0) = 0$ and $\phi_t(x,0) = 0$). The index i runs over the number N_L of points contained in the segment of length L. Here

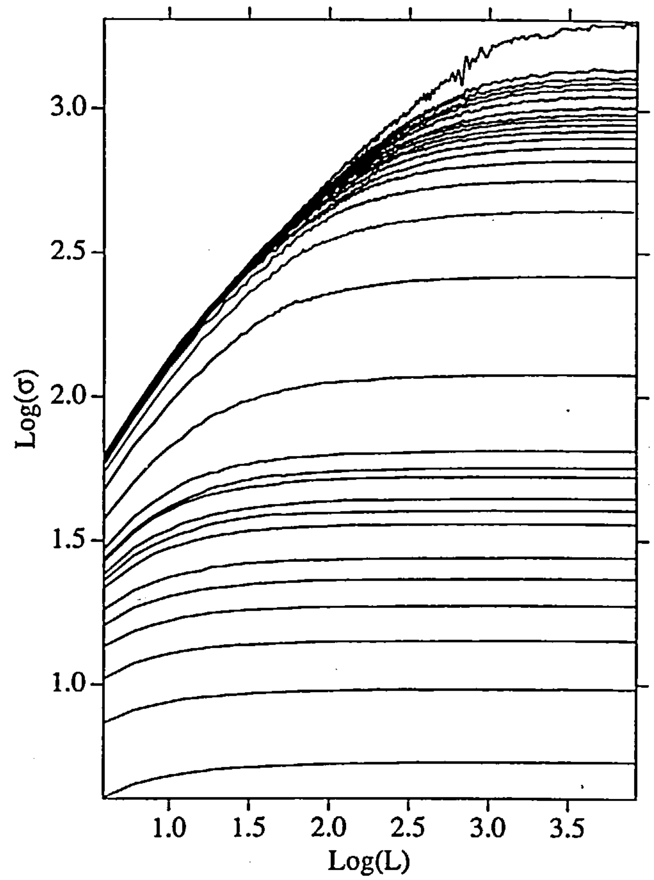

Fig. 1 Evolution towards the stationary regime of the standard deviation of the spatiotemporal profile ($\alpha = 0.252$). We discretized the equation into 8192 points for $l = 320$.

$\alpha = 0.252$ and the variance of the noise equals 3,333.33 (Ref. 3 presents curves for the onset of the solitonic regime as the variance of the noise is increased) and $\langle \ldots \rangle$ means ensemble average (we consider realizations from flat initial conditions in all the cases but the upper curve which corresponds to the stationary regime). We plot $\log(\sigma(L, t))$ versus $\log(L)$ for different times. The following features can be appreciated in Fig. 1: (1) the function $\sigma(L, t)$ for small length scales ($L \leq 1.25$) evolves towards a stationary state faster than for larger scales; and (2) the scaling of $\sigma(L, t)$ for very large times is such that the curve essentially do not grow any more (in the stationary regime, $\sigma(L) \equiv \sigma(L, t \to \infty)$) and approaches two different scaling exponents. Let us first analyze the stationary regime in which the time of computation is such that $t \gg t_c(L)$ and it is expected that $t_c(L)/L^z \cong 1$. Here z is the ratio ζ/β, between ζ (the roughening exponent) and the dynamic exponent β. We can call $t_c(L)$ the crossover time.[13]

The upper curve of Fig. 2 corresponds to the stationary regime for $\alpha = 0.252$; this curve reveals two different scaling behaviors $\sigma(L) \approx L^\zeta$, particularly, a KPZ behavior ($\zeta = 0.50$) for larger scales. For small length scales $\zeta = 0.69 \pm 0.01$ whereas $\zeta = 0.491 \pm 0.002$ for larger scales. For very large length scales a crossover to zero roughening exponent takes place. The crossover length for the scaling behavior depends on the value of the dissipation α and appears to be the same for different variances of the noise.[4]

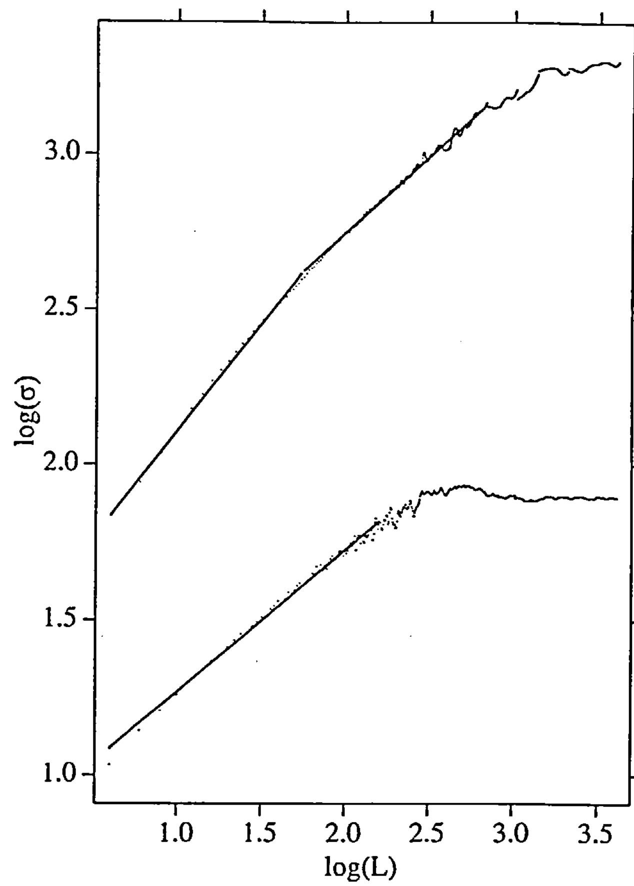

Fig. 2 *Stationary regimes* ($l = 160$). $\log \sigma$ vs. $\log L$ for $\alpha = 0.252$ (upper curve) and $\alpha = 25.2$ (lower curve); α is a measure of the damping. We present linear fits with slopes ~ 0.70 and ~ 0.50 for the upper curve; we also present fit with slope ~ 0.46 for the lower curve.

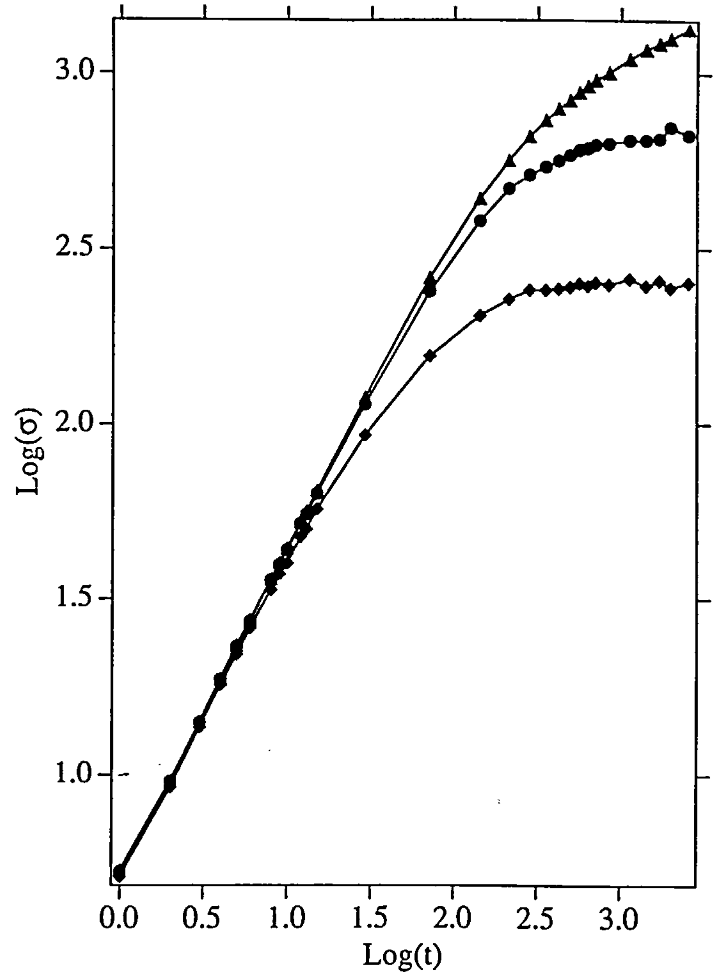

Fig. 3 *Transient regime* ($l = 320$). $\log \sigma$ vs. $\log t$ for $\alpha = 0.252$. The three different curves correspond (from below to above) to the three different values of the roughening exponent ($\zeta \sim 0.70$, $\zeta \sim 0.50$ and $\zeta \sim 0.0$).

The lower curve in Fig. 2 corresponds to the value $\alpha = 25.2$. In this case we find that the crossover length has shrunk to a negligible value and there is only one scaling behavior $\sigma(L) \approx L^{\zeta}$, with $\zeta = 0.457 \pm 0.004$.

In Fig. 3 we plot $\log(\sigma(L, t))$ versus $\log(t)$ for $\alpha = 0.252$. The three curves correspond to different lengths in the chain which are associated with different scalings in the stationary regime (i.e., $\zeta \cong 0.70$, $\zeta \cong 0.50$ and $\zeta \cong 0.0$). We find that for sufficiently small values of t, the three curves give a scaling $\sigma(t, L) \approx t^{\beta}$ with $\beta \cong 0.90$. As the system evolves, the curves begin to separate and for the shortest lengths the stationary regime is reached early. For L corresponding to the roughness exponent $\zeta \cong 0.0$ in the stationary regime, the dynamic exponent exhibits a crossover from $\beta \cong 0.90$ to $\beta \approx 0.283 \pm 0.008$.

Figure 4 is the same kind of plot as Fig. 3 but for $\alpha = 25.2$. We obtain a scaling with $\beta \cong 0.40$ for the initial times. Consistently, the scaling with $\beta \approx 0.90$, is absent. For the larger L, the upper curve reveals that there is another linear regime with $\beta \approx 0.2527 \pm 0.0004$. Both values are around the KPZ value $\beta = 1/3$. These results can be explained as in the overdamped limit ($\alpha \to \infty$) we expect to have only the case with $\beta = 1/3$.

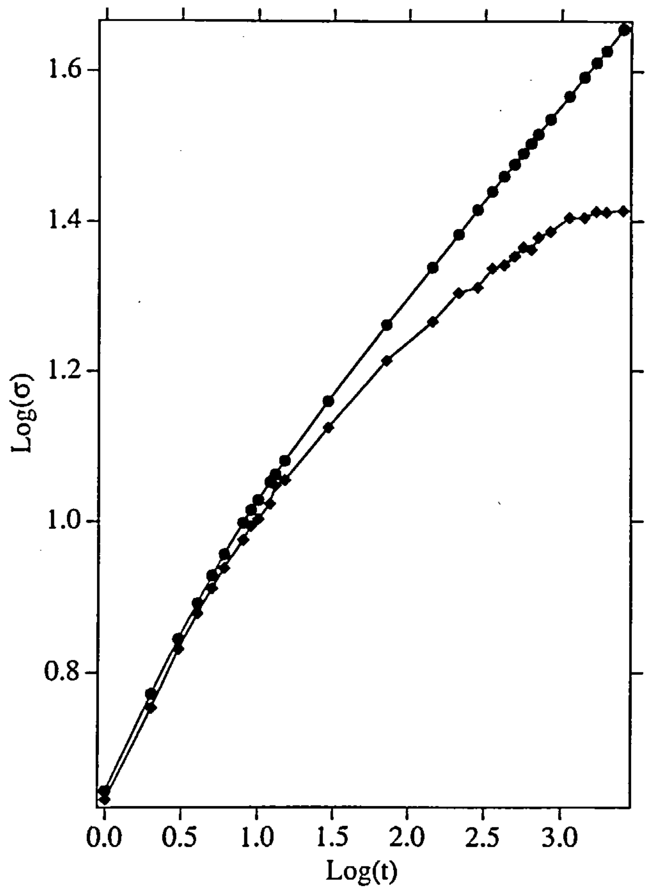

Fig. 4 *Transient regime* (l = 320). Log σ vs. log t for the overdamped regime (α = 25.2). Curves correspond (from below to above) to the two different values of the roughening exponent ($\zeta \sim 0.46$ and $\zeta \sim 0.0$).

4. CONCLUDING REMARKS

There has been some controversy in the literature concerning the random and damped Sine-Gordon model in the overdamped limit. Rost and Spohn made an enlarged Nozières-Gallet renormalization analysis,[14] and concluded that the nonlinear term in the KPZ equation is indeed generated but for the case when the driving force $F \neq 0$. All these analyses concern the case in which the pinning force is small, i.e., $V_0 \ll 1$. Therefore KPZ behavior should be present for sufficiently large scales. The calculations of Krug and Spohn[11] for the case of low levels of noise and a dc driving force $F \neq 0$, also give KPZ behavior with a temporal scaling given by $\beta = 1/3$. Our results confirms this picture also for the case of zero dc driving force and sufficiently high levels of noise and for the large scale behavior. However, the new scaling regime is different, with new scaling exponents and the existence of a crossover length within which the anomalous exponents exist. We call the non-KPZ exponents anomalous. We ascribe these new exponents which are bigger than the KPZ exponents to the coherence that the solitons maintain statistically along the crossover length l_c. Within this length there is global behavior, whereas solitons separated for length larger than l_c behave like a kink gas, with each one acting individually; at these scales the dynamics is local. It is interesting that this qualitative difference also makes the point in the Sneppen model in which dynamics is just global. The Sneppen model gives rise to an exponent $\beta = 0.9$, similar to ours. However, his roughening exponent, $\zeta = 0.63$ is quite different than our exponent $\zeta = 0.7$. We could consider our exponents as belonging to a new universality class: the literature reports experiments in which an anomalous roughening exponent is measured and coincides with our number.[7,15]

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