EUROPHYSICS LETTERS 1 June 2001
*Europhys. Lett.*, **54** (5), pp. 647–653 (2001)

# Non-ergodicity and fluctuations in mesoscopic insulators: The replica cooperon and diffuson

E. Medina[1], E. Roman[1] and R. Rangel[2]

[1] *Laboratorio de Física Estadística de Sistemas Desordenados, Centro de Física, IVIC Apartado 21827, Caracas 1020A, Venezuela*
[2] *Departamento de Física, Universidad Simón Bolívar Apartado 89000, Caracas 1080A, Venezuela*

(received 25 September 2000; accepted in final form 16 March 2001)

PACS. 72.20.-i – Conductivity phenomena in semiconductors and insulators.
PACS. 73.50.Dn – Low-field transport and mobility; piezoresistance.
PACS. 74.40.+k – Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.).

**Abstract.** – We explore the mesoscopic conductance fluctuations in the insulating regime within the Nguyen, Spivak, and Shklovskii model. We find that fluctuations of the log-conductance are persistent above the decorrelation field $B_c$ in the insulating regime. Using the replica approach, we derive the field coupling and fluctuations in terms of "cooperon" and "diffuson" analogs and determine new corrections to temperature dependencies for small $\Delta B$. We also analyze the ergodicity of fluctuations in the log-conductance and its scaling properties, and discuss the asymptotic validity of the usual criterion involving the commutability of disorder and field fluctuation averages.

Mesoscopic quantum effects, in the deep insulating regime (DIR), have been amply observed in a variety of materials [1–4]. The DIR is defined as that regime where the localization length is the smallest scale compared to the elastic mean free path and hopping lengths, *i.e.* $\xi < \ell < t$, respectively [5]. Coherence effects are possible in this regime because phase-breaking events occur at the hopping length [6], which is larger than $\ell$. The most striking signatures of quantum interference in disordered insulators are the classic magneto-fingerprints, or reproducible fluctuations in the conductance with magnetic field, and a low-field positive magneto-conductance.

An important property of mesoscopic conductance fluctuations in the metallic range is their ergodicity. Ergodicity, in the sense of Lee and Stone [7] (LS) is defined as the equivalence of sample-to-sample fluctuations and fluctuations with respect to field (or energy) in a single sample. More rigorously, given a physical quantity $F(\mathcal{H}, B)$ depending on the disordered Hamiltonian $\mathcal{H}$ and magnetic field $B$, one casts the ergodicity condition as $\overline{[\overline{F(\mathcal{H},B)} - \langle F(\mathcal{H},B)\rangle]^2} \to 0$ [8] and simultaneously $\overline{F(\mathcal{H},B)} = \overline{\langle F(\mathcal{H},B)\rangle}$. The overbar denotes ensemble average or average over disorder Hamiltonians and the angular brackets, a running average defined by

$\langle F(\mathcal{H}, B)\rangle = \Delta^{-1}\int_{B-\Delta}^{B+\Delta} \mathrm{d}B\ F(\mathcal{H}, B)$. The verification of both conditions is known as *ergodicity in the mean square limit* [8]. Such definition of ergodicity is consistent with its use in statistical mechanics, and has been used recently in the context of random matrix theory also involving energy averages [8].

Log-conductance fluctuations in the DIR, as opposed to the metallic phase, are not ergodic in the sense of LS, as has been shown experimentally [1, 3, 9], *i.e.* the variance over samples is larger than the variance over field. Such samples involve hopping lengths that are, at most, 6 to 10 times the localization length. Convincing results of Orlov *et al.* [9] and Ladieu *et al.* [3] have shown that a) field fluctuations do not decorrelate disorder fluctuations, and b) field fluctuations do not change the identity of the hop. On the other hand, there is a decorrelation field $B_c$, beyond which field fluctuations are independent [10, 11]. Surprisingly, the fluctuations beyond this field scale do not correspond to that of a new sample as can be derived from conclusion a).

On the theoretical side, the Nguyen, Spivak, and Shklovskii [5,6] model (NSS) has been successful in predicting LS non-ergodicity as opposed to random matrix [12] and Anderson models [13] of the insulating regime. The crucial insight of the NSS model is that coherence is maintained within a Mott hopping length [6], where the conductance is a sum of coherent *forward-directed Feynman paths* which interfere. The NSS model describes the quantum behavior of the critical (bottleneck) hop in the Miller-Abrahams network [14]. The existence of many critical hops tends to average the macroscopic conductance, eliminating fluctuations [10]. Here, we focus on the low-temperature regime where critical hops do not trivially self-average [15], *i.e.* so the percolation correlation length $\xi_\mathrm{p}$ is such that $\xi_\mathrm{p} = \xi(T_0/T)^{(\nu+1)/(D+1)} \sim L$, where $\nu$ is the percolation correlation length exponent, $D$ is the spatial dimension, and $T_0$ a disorder parameter.

In the present study, we propose to examine the fluctuations in the DIR in two dimensions, as predicted by the *asymptotic properties* of the NSS model. We will first show that there are two fluctuation regimes; one that responds to the absolute value of the magnetic field and the other, only responding to field differences. The two regimes are analogous to those of mesoscopic metals, but are yet undiscussed in the DIR. We then check the scaling behavior of the field correlation function and compare these results with those of the mean-square definition of ergodicity. The results reveal the fulfilment only of the condition on the first moment. Within the replica moment method [16], we derive a relation between the log-conductance correlation function, with field and temperature, and derive the replica cooperon and diffuson.

In the two-dimensional NSS model, impurities are placed on the sites of a lattice of main diagonal length $t$ (the hopping length). We apply a magnetic field $B$, perpendicular to the plane, changing only the phases of the electron paths. The overall tunneling amplitude is computed by summing all forward-directed paths between two diagonally opposed points, each contributing an appropriate quantum-mechanical complex weight given by the Hamiltonian

$$\mathcal{H} = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{\langle ij\rangle} V_{ij} a_i^\dagger a_j, \qquad (1)$$

where $\epsilon_i$ is the site energy, and $V_{ij}$ represents the nearest-neighbor couplings or transfer terms. Within the NSS model, we choose site energies to be $\epsilon_i = \pm W$ with equal probability. Following refs. [6] and [16], the Green's function between the initial and final site is given by

$$\langle i|G(E)|f\rangle = \left(\frac{V}{W}\right)^t J(B,t); \qquad J(B,t) = \sum_{\Gamma'}^{\text{directed}} \prod_{i_{\Gamma'}} \eta_{i_{\Gamma'}} e^{iA}, \qquad (2)$$

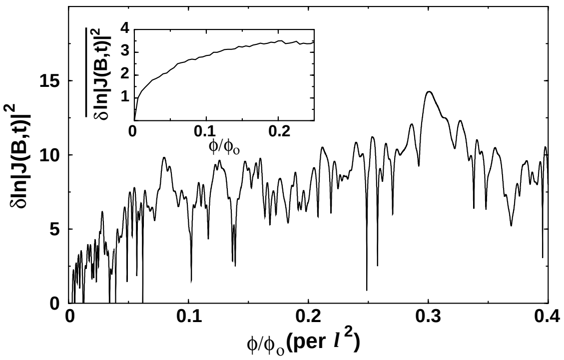


Fig. 1 – Conductance fluctuations $\delta \ln |J(B,t)|^2 = \ln |J(B,t)|^2 - \ln |J(0,t)|^2$ in the field for a single sample in two dimensions for $t = 62\ell$, where the average behavior is visible above the fluctuations. The inset shows the behavior of the average $\ln |J(B,t)|^2$ after 500 realizations.

where $A$ is the magnetic vector potential, $\Gamma'$ represents all directed paths that go from $i$ to $f$ through the lattice and $\eta_i = \text{sign}(\epsilon_i) = \pm 1$. The function $J(B,t)$ contains the interference information including correlations due to crossing of paths, and the factor $(V/W)^t$ is the leading contribution to the exponential decay of the localized wave function. We use the transfer matrix approach in order to compute $J(B,t)$, exactly, for each realization of disorder [16]. The effect of a disordered arrangement of impurities within the directed path model has been addressed in ref. [17], and is not relevant for the present study.

Figure 1 depicts typical fluctuations of the log-conductance for a fixed sample as a function of the magnetic field. The figure clearly shows that the average log-conductance dominates the fluctuations, *i.e.*, the average behavior is visible for a single sample. The inset to fig. 1 shows the average log-conductance first increasing proportional to $B$, crossing over to a slower growth as $B^{1/2}$ dictated by the magnetic length [16, 18]. In the latter regime, while the log-conductance tends to saturate, the fluctuations persist as in mesoscopic fluctuation theory in metals [7]. Furthermore, we note that the average behavior is periodic in half the flux quantum per $\ell^2$ (only one half of a period is shown). This periodicity reveals an average field coupling to $2B$ [6] which will be demonstrated theoretically below. In three dimensions, the fluctuations are appreciably larger than the average behavior. Once more, persistent fluctuations beyond the average conductance saturation field are observed. The existence of such persistent fluctuations was first surmised by Sivan *et al.* [10, 19] and Zhao *et al.* [18].

On the metallic side, an analogous behavior has been described, and identifies two fundamental contributions to the field effect: The cooperon and the diffuson [7]. These contributions can be distinguished by the way they enclose the magnetic flux; while the cooperon is sensitive to $2B + \Delta B$, the diffuson only responds to field changes $\Delta B$. In the insulating regime, a cooperon-like mechanism associated with a positive magneto-conductance (MC) which saturates, has already been observed as a general effect [1, 2, 4, 9]. On the other hand, as shown in fig. 1, conductance fluctuations persist in the insulating regime to relatively large fields, exposing a diffuson-like mechanism.

According to the condition of LS, ergodicity is assessed by comparing the magnitude of the variance in the field (or energy) and sample-to-sample fluctuations. Figure 2 shows the averages

$$\text{Var}_d(\ln |J(B,t)|^2) = \overline{(\ln |J(B,t)|^2 - \overline{\ln |J(B,t)|^2})^2}\,, \tag{3}$$

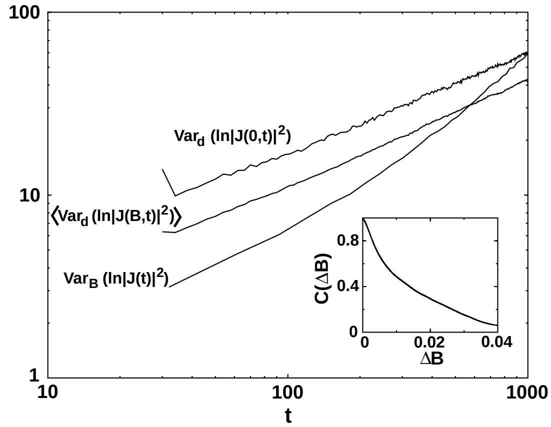


Fig. 2 – The figure depicts the variance as defined by eqs. (3) and (4). The sample-to-sample variance (also shown averaged in the field) is initially larger than the variance in the field, but the latter increases faster as a function of $t$ and eventually intersects the former. The inset shows the field correlation function $C(B,\Delta B,t) = \overline{(\Delta\ln|J(B+\Delta B,t)|^2)(\Delta\ln|J(B,t)|^2)}$, where $\Delta\ln|J(B,t)|^2 = \ln|J(B,t)|^2 - \overline{\ln|J(B,t)|^2}$.

and

$$\mathrm{Var}_B(\ln|J(B,t)|^2) = \overline{\langle(\ln|J(B,t)|^2 - \langle\ln|J(B,t)|^2\rangle)^2\rangle}\,. \qquad (4)$$

In the figure we show the variance in the field according to eq. (4), for fields above 0.01 flux quantums per $\ell^2$. Initially, $\mathrm{Var}_B(\ln|J(B,t)|^2)$ is smaller than $\mathrm{Var}_d(\ln|J(0,t)|^2)$ but the former increases faster with $t$ and eventually crosses the latter. Such crossing indicates that the system can become "ergodic" in the LS sense, as the hopping length increases. Obviously, this happens for $t/\xi$ much larger than that expected for experiments, and in real systems it only represents a tendency. A further problem with the comparison in the figure is that the field behavior is non-stationary, so the difference between the variances depends on $B$. Koslov *et al.* [9] avoided this issue (with experimental data) by further averaging $\mathrm{Var}_d(\ln|J(B,t)|^2)$ in the field. Figure 2 also shows the latter average, implying that fluctuations can become ergodic, according to LS criterion, at smaller hopping lengths. The inset of fig. 2 depicts the field correlation function (defined in the caption) showing a rapid exponential decay in contrast with the algebraic decay in the metallic regime [7].

Using now the mean-square limit definition of ergodicity, we obtain fig. 3, showing that the condition on the variance $\overline{[\overline{\ln|J(\mathcal{H},B,t)|^2} - \langle\ln|J(\mathcal{H},B,t)|^2\rangle]^2} \to 0$ is not met. The deviation from zero of the latter increases linearly with the size of the hop as indicated in the figure. On the other hand, the inset shows the asymptotic convergence, as averaging increases, to the condition $\overline{\ln|J(\mathcal{H},B,t)|^2} = \langle\ln|J(\mathcal{H},B,t)|^2\rangle$. Thus, according to the mean-square criterion, fluctuations in the localized regime meet only a weak ergodicity condition in the large-size limit. We have determined that our conclusions are relatively independent of the running average interval $\Delta$ as long as $(\Delta^2/24)\partial^2(\overline{\ln|J(B)|^2})/\partial B^2 < \overline{\ln|J(B)|^2}$ [8].

The cooperon-diffuson contributions in the previous results can be accounted for within the replica moment approach [16] as follows: The generator of the correlation function defined in fig. 2 is the moment $\overline{[J^*(B+\Delta B)J(B+\Delta B)J^*(B)J(B)]^n}$, where each $J(B)$ represents the sums of all single paths between the initial and final sites. In order to have nonzero contributions after disorder average, the paths must pair up [16]. Neutral paths (field independent)

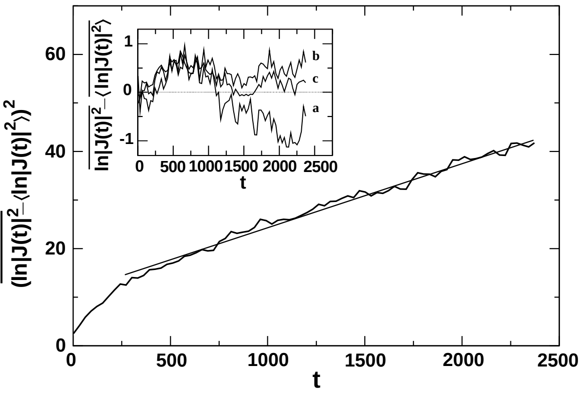


Fig. 3

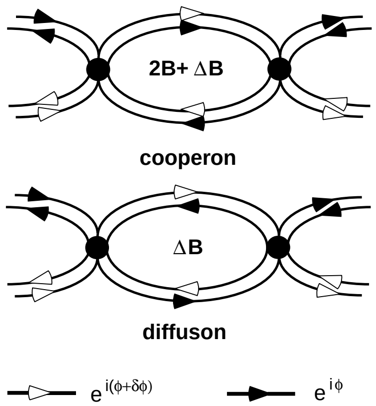


Fig. 4

Fig. 3 – Test of ergodicity in the mean-square limit as defined in the text. The figure depicts the condition on the variance showing no convergence to zero with either number of realizations or hopping length. The straight line is given as a guide to the eye. The inset shows the behavior of the condition on the average which does conform to the ergodic requirement asymptotically. Labels a, b, and c correspond to increasing number of realizations in disorder.

Fig. 4 – Replica cooperon and diffuson. Arrows corresponding to paths picking up phases of $\phi = \int \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$ and $\phi + \delta\phi = \int \boldsymbol{A} + \mathrm{d}\boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$ are shown.

are formed by pairing $J^*$ and $J$ at the same field (phase cancels). On the other hand, charged paths (field sensible) are formed by pairing either $J^*(B+\Delta B)$ and $J(B)$ or $J^*(B+\Delta B)$ and $J^*(B)$. In the absence of paired path intersections, self-interference kills charged paths (their contribution decays exponentially fast). Nevertheless, if intersections are considered, one can have path exchanges, for short distances, yielding a magnetic-field coupling which is the source of MC [16].

Figure 4 shows the two possible field-sensitive diagrams at a paired path crossing: a) One partner from $J^*(B+\Delta B)$ pairs with one from $J^*(B)$ while one from $J(B+\Delta B)$ and one from $J(B)$ follow a different path. Such a combination encloses $(2B+\Delta B)$, and is therefore called *cooperon*-like. b) One partner is taken from $J^*(B+\Delta B)$ and the other from $J(B)$ on the same path, while one from $J(B+\Delta B)$ and $J^*(B)$ follow another. Such a combination encloses only $\Delta B$ and is called *diffuson*-like. Note that all previous cases satisfy overall neutrality so that the contributions are real as expected. The contribution of the replica cooperon and diffuson are the same at zero field and there is an additional contribution from the uncharged bubble. Thus, when the cooperon is suppressed, one expects only a reduction of $\mathrm{Var}_d(\ln|J(0,t)|^2)$ by one third which is in agreement with our numerical results [16].

The replica moment argument, as applied in ref. [16], maps the $n$-th moment problem onto the problem of $2n$ bosons with contact interactions. These interactions renormalize due to the diagrams in fig. 4, making path interactions field dependent. The $2n$ boson system can be solved using the Bethe ansatz [20] and has ground-state energy $\epsilon_0 = \ln 4n + \rho(B,\Delta B)n(n^2-1)$. On the other hand, the $n$-th moment can be expressed as a cumulant expansion, and identifying orders of $n$ we arrive at the expressions

$$\mathrm{Var}_d(\ln|J(B+\Delta B)|^2)+\mathrm{Var}_d(\ln|J(B)|^2)+C(B,\Delta B) = (\rho_{\mathrm{coop}}(2B+\Delta B)+\rho_{\mathrm{diff}}(\Delta B))t^{2/3}. \quad (5)$$

Here we have separated the path interaction in terms of the cooperon and diffuson contributions. Beyond the saturation field of the average log-conductance, the cooperon term on the right-hand side saturates (the same for the variances on the left which depend on $B$) and the correlation function only depends on $\Delta B$. The field dependence of the right of eq. (5) is $\rho_{\rm coop}(B,\Delta B)+\rho_{\rm diff}(\Delta B) = \cos(\frac{2B+\Delta B}{B_c})+\cos(\frac{\Delta B}{B_c})+1$. The replica results are valid quantitatively for $1+1$ dimensions, nevertheless, $2+1$ dimensions also have a bound state, although weaker, with a smaller positive MC. Therefore, our results apply qualitatively to three-dimensional hops.

From the dependence of $\rho(B,\Delta B)$, one can compute the full temperature dependence of $C(B,\Delta B)$, for small $\Delta B$, by expanding the field dependence of the diffuson part such that $\Delta B < B_c = \pi c\hbar/(\xi^{1/2}e\ t^{3/2})$ [6] (the NSS decorrelation field):

$$C(\Delta B)=\rho(B)\xi\left(\frac{T_0}{T}\right)^{2/9}\left(1-\frac{1}{2}\left(\frac{\Delta B\xi^{3/2}e}{\pi\hbar c}\right)^2\left(\frac{T_0}{T}\right)^{4/9}\right)\qquad\text{for}\qquad \Delta B<B_c, \tag{6}$$

where $T_0=\Delta/N\xi^3$, $\Delta$ is the impurity band width and $N$ the impurity concentration. In the previous expression we have employed the temperature dependence of the $\Delta B=0$ fluctuations [16] using $\mathrm{Var}_d(\ln|J(B,t)|^2)=\rho(B)t^{2/3}$. The initial decay of the correlation function (see fig. 2) starts out as a plateau and then decays faster, in agreement with cosine expansion in eq. (6).

In order to experimentally observe persistent diffuson fluctuations, one has to explore a range of parameters so that there is a saturation in the average behavior while the wave function shrinkage [14] has not yet kicked in. This range can be defined by the condition $B_c < \hbar/(ea_{\rm B}N^{1/3}) = B_{\rm orb}$, where $a_{\rm B}$ is the Bohr radius. $B_{\rm orb}$ is the scale for the orbital shrinkage to be important [3], *i.e.*, when the cyclotron radius becomes of the order of the mean free path $\ell$. These conditions have been met in ref. [9] and [3]. Furthermore, according to refs. [3, 9, 21], the magnetic field cannot induce geometric fluctuations due to changes in the identity of the hop [22]. If the latter is true in two and three dimensions, the insulating diffuson fluctuations should be seen experimentally.

Summarizing, we have assessed the field fluctuations and correlations of the logconductance in the strongly localized regime within the NSS model. Persistent fluctuations at fields beyond $B_c$ are observed and explained using a replica diffuson and cooperon picture. We compared the variances in sample-to-sample and field fluctuations and found that insulators become increasingly ergodic, according to the Lee and Stone criterion, as the hopping length increases. A more stringent definition of ergodicity, in the mean-square limit, shows that the NSS model only meets a weak condition on the average while the variance criterion is not satisfied for any hopping length. The full temperature dependence of the fluctuation correlation function, at small $\Delta B$, is obtained beyond the field scale at which the cooperon is suppressed. Analytical treatment of fluctuation correlations should be possible using the percolation mean field [19] and other uncorrelated path approaches [23, 24]. Such results could shed light on the intrinsic non-ergodic nature of the directed path model. We emphasize that the present study assesses the asymptotic properties of the NSS model and the relation between the localization length and the hopping length considered have not been realized experimentally. Finally, we must point out that as temperature increases, other mechanisms which increase ergodicity (such as returning paths) will arise that cannot be accounted for within the NSS model. These contributions can only be studied with more general Green's function approach.

* * *

This work was supported by CONICIT through grant G97-000670.

REFERENCES


[1] LAIKO E. I., ORLOV A. O., SAVCHENKOV A. K., IL'ICHEV E. A. and POLTORATSKII E. A., *Zh. Eksp. Teor. Fiz.*, **93** (1987) 2204; (*Sov. Phys. JETP*, **66** (1987) 1258).

[2] ZHANG Y., DAI P. and SARACHIK M. P., *Phys. Rev. B*, **45** (1992) 9473.

[3] LADIEU F., MAILLY D. and SANQUER M., *J. Phys. I*, **3** (1993) 2321; LADIEU F. and BOUCHAUD J. P., *J. Phys. I*, **3** (1993) 2311.

[4] FARAN O. and OVADYAHU Z., *Phys. Rev. B*, **38** (1998) 5457.

[5] SHKLOVSKII B. I. and SPIVAK B. Z., *Hopping Transport in Solids*, edited by POLLAK M. and SHKLOVSKII B., Vol. **28** (Elsevier Science Publishers B. V., Amsterdam) 1991.

[6] NGUYEN V. L., SPIVAK B. Z. and SHKLOVSKII B. I., *JETP Lett.*, **41** (1985) 42; *Sov. Phys. JETP*, **62** (1985) 1021; *JETP Lett.*, **43** (1986) 44.

[7] LEE P. A. and STONE A. D., *Phys. Rev. Lett.*, **55** (1985) 1922; ALTHSULER B. L. and KHMEL'NITSKII D. E., *JETP Lett.*, **42** (1985) 359; LEE P. A., STONE A. D. and FUKUYAMA H., *Phys. Rev. B*, **35** (1987) 1039.

[8] PANDEY A., *Ann. Phys. (N.Y.)*, **119** (1979) 170; PLUHAR Z. and WEIDENMULLER H. A., *Phys. Rev. Lett.*, **84** (2000) 2833.

[9] ORLOV A. O., SAVCHENKO A. K. and KOSLOV A., *Solid State Commun.*, **72** (1989) 743.

[10] FOWLER A. B., WAINER J. J. and WEBB R. A., *Hopping Transport in Solids*, edited by M. POLLAK and B. I. SHKLOVSKII, *Modern Problems in Condensed Matter Sciences*, Vol. **28** (North Holland, Amsterdan) 1991, p. 233.

[11] MILLIKEN F. P. and OVADYAHU Z., *Phys. Rev. Lett.*, **65** (1990) 911.

[12] PICHARD J. L., SANQUER M., SLEVIN K. and DEBRAY P., *Phys. Rev. Lett.*, **65** (1991) 1812; AVISHAI Y., PICHARD J. L. and MUTTALIB K. A., *J. Phys. I*, **3** (1993) 2342.

[13] FENG S. and PICHARD J. L., *Phys. Rev. Lett.*, **67** (1991) 753.

[14] SHKLOVSKII B. I. and EFROS A. L., *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin) 1984.

[15] LADIEU F. and SANQUER M., *Ann. Phys. (Paris)*, **21** (1996) 267.

[16] MEDINA E. and KARDAR M., *Phys. Rev. B*, **46** (1992) 9984.

[17] BALENTS L. and KARDAR M., *J. Stat. Phys.*, **67** (1992) 1.

[18] ZHAO H. L., SPIVAK B. Z., GELFAND M. P. and FENG S., *Phys. Rev. B*, **44** (1991) 10760.

[19] SIVAN U., ENTIN-WOHLMAN O. and IMRY Y., *Phys. Rev. Lett.*, **60** (1998) 1566; *Phys. Rev. B*, **40** (1989) 8342.

[20] KARDAR M., *Nucl. Phys. B*, **290** (1987) 582.

[21] LADIEU F. and BOUCHAUD J. P., *J. Phys. I*, **3** (1993) 2311.

[22] LEE P. A., *Phys. Rev. Lett.*, **53** (1984) 2042.

[23] LIN Y.-L. and NORI F., *Phys. Rev. Lett.*, **76** (1996) 4580; *Phys. Rev. B*, **53** (1996) 15543.

[24] RAIKH M. E. and RUZIN I. M., *JETP Lett.*, **43** (1986) 562.